Vortex-Induced Lift and Drag on Stationary and Vibrating Bluff Bodies

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Several writers have recently proposed to predict the unsteady forces on and flowfields about stationary and vibrating bluff bodies by approximating the flowfield with the classical formulations for the infinite and semi-infinite von Kármán streets of staggered vortexes. This paper discusses the validity of certain of these models and, in particular, the suitability of such models when the body is vibrating. Some new experimental results for the vortex strength and spacing in the wakes of vibrating circular cylinders are employed here to compute the lift coefficients which result from some proposed prediction models which are based on the von Kármán street formulation. The various contributions to the steady drag are also discussed. It is shown, based on some recent experimental results, that the drag on stationary and vibrating bluff bodies is predominantly vortex-induced. The von Kármán drag coefficients obtained from wake measurements are in good agreement with direct force measurements which were made on vibrating circular cylinders at Reynolds numbers between 80 and 4000.

Nomenclature

a	= cylinder amplitude of transverse vibration, peak-to-peak, m
C_D	= drag coefficient
C_{DK}	= von Kármán drag coefficient, Eq. (11)
C_L	= lift coefficient
d	= cylinder diameter, m
f	= cylinder vibration frequency, Hz
f_s	= Strouhal frequency of vortex shedding from a stationary cylinder
F	= fluid force on a bluff body, Eq. (1), N
h	= lateral spacing of vortexes, m
J	= momentum contained within a control volume, Eq. (1), $kg\ m/s$
ℓ	= longitudinal vortex spacing, m
P	= integrated pressure over the surface of a control volume, Eq. (1), N
Re	= Reynolds number, Ud/ν
S	= flux of momentum through a control surface, Eq. (1), N
St	= Strouhal number for vortex shedding from a stationary cylinder, $f_s d/u$
St^*	= wake capture Strouhal number for a vibrating cylinder, $f(a+d)/U$
u_s	= induced velocity for a street of vortexes, m/s
U	= incident flow velocity, m/s
Γ	= initial vortex circulation, Eq. (2), m^2/s
ρ	= fluid density, kg/m^3
ν	= fluid kinematic viscosity, m^2/s

Introduction

A RECENT paper[1] which deals with the determination of the periodic lift force on a bluff body and a later discussion of the paper[2] deserve some further comments and clarifications. The topics to be discussed here are the general validity of a proposed model[1] for predicting the unsteady forces on bluff bodies, and the suitability of that model when the body is vibrating. Another point which deserves consideration is the suitability of some empirical relationships which have been suggested to determine the periodic lift force on a bluff body in terms of experimental results for the drag coefficient, Strouhal number, and vortex spacing. A point raised in a recent paper[2] with regard to the various contributions to the total drag is also discussed. It is shown, on the basis of recently published results,[3-6] that the drag on stationary and vibrating bluff bodies is predominantly vortex-induced.

Some other writers[7-8] have also recently attempted to predict the forces on and flowfields about stationary and vibrating bluff bodies and airfoils by approximating the flowfield with the classical expressions for the infinite and semi-infinite von Kármán vortex streets. A thorough exposition of these particular vortex configurations has been given by Synge,[9] Milne-Thomson,[10] and Kochin, Kibel and Roze.[11]

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The von Karman Models for the Lift and Drag

The use of the fluid momentum equation, integrated over an appropriate control volume, was first employed by von Kármán (see Milne-Thomson[10] or Kochin et al.[11]) and Synge[9] to determine the steady drag on a bluff body. The momentum equation in its integral form can be written for an arbitrary control volume as

$$P_{x,y} = (dJ_{x,y}/dt) + S_{x,y} + F_{x,y} \tag{1}$$

where the subscripts x and y in this case denote directions parallel and perpendicular to the incident freestream direction, and where F is the fluid force on the cylinder, P is the pressure force on the control surface, S is the momentum flux through the control surface, and J is the momentum combined within the control volume at any time t.

The theoretical equation for the von Kármán drag coefficient C_{DK} agrees well with experiments, both for stationary[11] and vibrating circular cylinders.[4] Since in computing the steady drag the momentum equation is integrated both spatially over a control volume and temporally over a period of the vortex shedding, it is not surprising that good agreement is obtained with such a gross feature of the system. The insensitivity of the steady drag to the details of the flowfield becomes evident when one considers that both von Kármán and Synge obtained the same drag equation even though they respectively assumed the vortex street to be infinite and semi-infinite in extent. However, the use of the integral momentum equation is more precarious when an instantaneous feature of the system such as the lift is computed using the methods introduced by Sallet[1] and Chen.[7] The periodic lift force is closely dependent on the temporal and spatial variations of the flow and pressure fields near the body, and is less likely to be amenable to integral formulations which do not account for details of the flowfield.

Several further assumptions are necessary to employ Eq. (1), since, as noted by Batchelor,[12] it is often impossible to solve the momentum equation because the volume integral J of the momentum in the control volume cannot be specified. Chen,[7] for example, has assumed that the momentum J_y contained at any time within the control volume is zero identically. As pointed out by Iwan & Blevins,[13] this neglects the momentum components of the near wake which determine the forces on the body, and yields the incorrect result that these forces are independent of the body cross-section. Sallet,[1] on the other hand, has postulated that each of the vortexes in the street is a part of two vortex pairs, thereby resulting in an expression of

$$J_y = \rho(\ell\Gamma/4) \tag{2}$$

where ρ is the fluid density, ℓ is the longitudinal spacing between vortexes of like sign, and Γ is the total circulation of a single vortex. These two approaches lead to the following expressions for the lift coefficient C_L.

Sallet[1]

$$C_L = \tfrac{1}{2}\,\frac{\Gamma}{Ud}\left[1 - 3\,\frac{u_s}{U}\right] \tag{3}$$

Chen[7]

$$C_L = 2\,\frac{\Gamma}{Ud}\left[\frac{u_s}{U}\right] \tag{4}$$

In these equations u_s is the induced velocity of the vortex street, U is the velocity of the freestream, and d is the cylinder's diameter. Both of these equations, regardless of the validity of the assumptions made in their derivation, depend either on the direct measurement of the vortex circulation, convection speed, and spacing or further empirical or ad hoc assumptions to specify the hydrodynamic properties of the wake as a function of the Reynolds number. These equations also do not take account of the three-dimensional character of and the lack of coherence in the near wake behind a stationary bluff body, as Payne[2] has correctly pointed out.

The Lift Forces on Vibrating Cylinders

The von Karman model has recently been proposed[1] as a means for predicting the lift forces on vibrating cylinders. This extension has a certain appeal because of the practical importance of determining the flow-induced forces on vibrating structures and because the wake behind a vibrating cylinder is more nearly two-dimensional than is the wake of a stationary body. As the amplitude of vibration is increased beyond a threshold value, the vortex frequency locks into or is captured by the cylinder frequency, the correlation and coherence of the vortex shedding along the body's length are greatly increased, and the wake becomes more nearly two-dimensional[3,6,14-17]. This wake capture or lock-in occurs not only when a flexible or elastically-mounted cylinder is resonantly excited through the action of fluid forces, but also when a cylinder is forced to vibrate at frequencies near the Strouhal frequency of vortex shedding from a stationary cylinder. Certain of the relationships proposed for use in connection with the von Karman model are now considered in light of some experimental findings which have just recently become available.[3-6,15]

The final equations for the fluid forces developed in Ref. 1 should be employed with caution in the case of a vibrating cylinder. After some manipulation, the following equations were obtained for the lift and drag coefficients

$$C_L = \frac{\ell}{d}\left[1 - St\,\frac{\ell}{d}\right]\left[3\,St\,\frac{\ell}{d} - 2\right]\coth\frac{\pi h}{\ell} \tag{5}$$

$$C_D = \frac{4\ell}{d}\left[\frac{h}{\ell}\left(1 - St\,\frac{\ell}{d}\right)\left(2\,St\,\frac{\ell}{d} - 1\right)\right.$$

$$\left.\times\coth\frac{\pi h}{\ell} + \frac{1}{\pi}\left(1 - St\,\frac{\ell}{d}\right)^2\times\coth^2\frac{\pi h}{\ell}\right] \tag{6}$$

where h is the lateral spacing of the vortex street and St is the Strouhal number. These two equations result when the additional relations

$$5u_s/U = (\Gamma/2U\ell)\,\tanh(\pi h/\ell) \tag{7a}$$

and

$$u_s/U = 1 - St(\ell/d) \tag{7b}$$

are utilized in connection with Eq. (3) and Eq. (11). The latter equation for the drag is described in the next section. If von Kármán's theoretical stability criterion is assumed to give the true vortex spacing, as has often been done, then

$$\cosh = (\pi h/\ell) = \sqrt{2}$$

or

$$(h/\ell) \sim 0.281 \tag{7c}$$

The von Kármán spacing ratio further reduces Eqs. (5-6) to the forms obtained in the literature,[1] or

$$C_L = \sqrt{2}\,\frac{\ell}{d}\left(1 - St\,\frac{\ell}{d}\right)\left(3\,St\,\frac{\ell}{d} - 2\right) \tag{8}$$

and

$$St^2\left(\frac{\ell}{d}\right)^3 + 0.529\,St\left(\frac{\ell}{d}\right)^2$$

$$-1.529\left(\frac{\ell}{d}\right) + 1.593\,C_D = 0 \tag{9}$$

It was first demonstrated by Koopmann[16] that the lateral vortex spacing h decreases with increasing amplitude of cylinder vibration. This behavior has recently been investigated in more detail, both by means of hot-wire measurements and flow visualization, by Griffin & Votaw[17] and Griffin & Ramberg.[3] It has also been shown in these same experiments that the longitudinal spacing ℓ remains constant with changes in amplitude at any given frequency, but that the longitudinal spacing varies inversely with frequency within the lock-in regime. A typical example is given in Fig. 1 wherein photographs of the wake of a cylinder are shown for several amplitudes of the cylinder's vibration at 90% of the Strouhal frequency, at a Reynolds number of 190. The longitudinal spacing remains constant under these changes in amplitude while the lateral spacing becomes smaller as the amplitude of vibration is increased. It has also been shown that the drag increases with both frequency and amplitude of cylinder vibration,[5,6] and that the increased drag is accompanied by changes not only in the spacing but also by changes in the strength of the vortexes (see Table 1). Neither Eq. (8) or (9) is appropriate for predicting the forces on and the vortex spacing near vibrating structures, as recently done in the literature,[1] since in the derivation of these equations it was

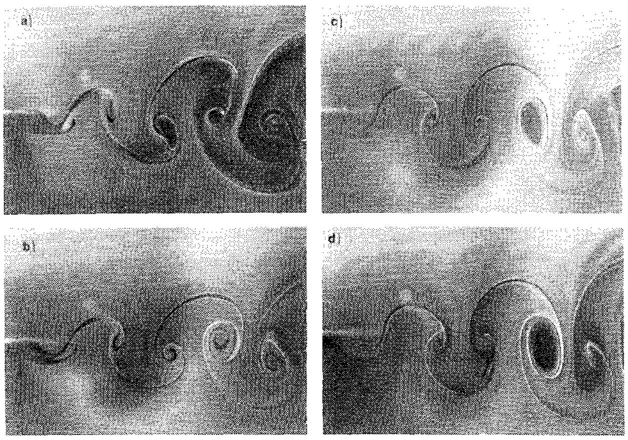

Fig. 1 Flow visualization of the vortex wake behind a vibrating cylinder. The vibration and vortex frequencies are locked together at 90% of the Strouhal frequency corresponding to the incident flow velocity. Reynolds number = 190; frequency, 32.9 Hz (all figures). Cylinder amplitude, peak-to-peak: a) 20% diam; b) 50% diam; c) 80% diam; d) 100% diam.

Table 1 Vortex drag on stationary and vibrating bluff bodies

St^*	ℓ/d	h/ℓ	$2(f\ell/U)-1$	$\Gamma/\pi Ud$	C_{DK}	$C_{DK}/C_{DK,0}$
a) Vibrating cylinder, Reynolds number = 144						
0.178[b]	5.4	0.180	0.9	0.81	1.21	1.00
0.199	5.4	0.156		1.11	1.70	1.40
0.208	5.9	0.126		1.27	1.76	1.46
0.231	5.4	0.134		1.23	1.81	1.50
0.254	4.9	0.146		1.33	2.23	1.84
0.263	5.4	0.112		1.35	1.92	1.58
b) Vibrating cable, Reynolds number = 450						
0.208[b]	5.0	0.180	0.88	0.64	0.90	1.00
0.270[a]	5.0	0.140		1.04	1.49	1.66

[a] Measurements made downstream from the cable antinode.
[b] Stationary cable and cylinder reference values.

assumed that the spacing ratio h/ℓ of the vortexes was equal to the von Kármán value, i.e., that $h/\ell = 0.281$. Moreover, although Eq. (8) will predict a change in the lift as a result of change in the longitudinal spacing ℓ with frequency, no change in the lift with amplitude can be predicted. The change in lift coefficient with amplitude is well-known and has been measured and reported in the literature.[5,18,19].

The vortex strength, spacing, and drag in the wake of a vibrating cylinder recently have been obtained and reported by Griffin and Ramberg.[3,4] In these papers a model for the vortex street was matched with velocity profiles which were measured at Reynolds numbers of 144 and 450. The results were obtained with no a priori assumptions other than that the solution was to satisfy a mean-square error criterion in the stable region of the vortex wake. A similar method was employed by Bloor and Gerrard[20] in their determination of turbulent vortex strength in the wake of a stationary cylinder. Table 1 contains the salient results of these experiments, including the drag coefficient C_{DK} obtained with the equation derived by Synge[9] and von Kármán.[10,11] The longitudinal spacing ℓ was measured directly, and the subsequent matching resulted in the initial circulation of the vortexes Γ, the lateral spacing h, and the viscous core radius r^* of the individual Hamel-Oseen vortexes. The parameter St^* is a "wake capture" Strouhal number

$$St^* = f(a+d)/U = St(1+\frac{a}{d})\ (f/f_s)$$

where f is the vibration (and vortex) frequency and a is the peak-to-peak amplitude of cylinder motion. The drag coefficient results included in Table 1 are discussed in more detail in the next section.

Table 2 The predicted von Kármán lift coefficients on stationary and vibrating cylinders

St	a/d	f/f_s	ℓ/d	h/ℓ	$\Gamma/\pi Ud$	C_L Eq. (5)	C_L Eq. (3)
0.178[b]	0.0[b]	1.0[b]	5.4[b]	0.18[b]	0.81[b]	0.37[a]	1.11[a]
0.199	0.12	1.0	5.4	0.16	1.11	0.41	1.53
0.208	0.30	0.9	5.9	0.12	1.27	0.58	1.76
0.231	0.30	1.0	5.4	0.13	1.23	0.49	1.70
0.254	0.30	1.1	4.9	0.14	1.33	0.42	1.84
0.263	0.48	1.0	5.4	0.11	1.35	0.57	1.87

[a] Lift coefficients predicted from the results of Sallet.[1]
[b] Experimental results for vortex circulation and spacing from the results of Griffin & Ramberg[3] for stationary and vibrating cylinders.

These results just mentioned, and others reported in the literature,[3,4,15-17] show that both the circulation Γ and the vortex spacings h and ℓ are strongly influenced by the vibrations. In view of these recent findings, it is instructive to compare the predictions which result from different proposed forms of the lift equation. Chaplin[21] has shown that the maximum lift amplitude is reduced when the vortex street is composed of Hamel-Oseen viscous vortexes. The results published by Griffin and Ramberg[3] take the finite core radii of the vortexes into account, and so the values of the initial circulation Γ listed in Table 2 can be interpreted as the vortex strength or circulation of an inviscid street of vortexes with the same initial value of circulation as the corresponding viscous vortex street. The viscous correction in the case of Griffin and Ramberg's experiments reduces the peak lift computation by only about 10%.

The lift coefficients C_L predicted with Eqs. (5) and (3) are compared in Table 2. Experimental values of vortex circulation and spacing from Table 1 have been used in the calculations, and the two equations yield quite different results. Both equations predict increased lift as the amplitude of vibration is increased, but the magnitudes of the lift coefficients obtained from the two equations are very different. These differences are due to the measured values of the induced velocity, $u_s/U = 0.05$-0.06, which deviate substantially from the theoretical value of $u_s/U = 0.14$. Most experimental results for the induced velocity u_s of finite vortex streets behind cylinders are considerably less than this theoretical value given by Eq. (7a). The two equations also yield different predictions for the effects of vibration frequency on the lift coefficient C_L—a decrease in lift with frequency is predicted by Eq. (5) and an increase in lift is predicted by Eq. (3)—when the experimental data listed in the table are used. Again, the magnitudes of the lift coefficients are quite different when vortex strength or spacing results are employed in the lift computations. Thus, whatever the merits of the assumptions inherent in the development of these equations, the use of the momentum integral, Eq. (1) for the prediction of unsteady forces on bluff bodies should be approached with caution.

Vortex-Induced Drag at Subcritical Reynolds Numbers

Payne[2] has suggested that the drag coefficient C_D on a bluff body at subcritical Reynolds numbers is made up of three contributions, or that

$$C_D = \Delta C_{D_{\text{skin friction}}} + \Delta C_{D_{\text{von Kármán}}} + \Delta C_{D_{\text{other}}} \tag{10}$$

and, further, that

$$\Delta C_{D_{\text{von Kármán}}} \to 0$$

as the Reynolds number approaches $R = 50$-100, the lower limit of the regime of vortex street formation. Payne has fur-

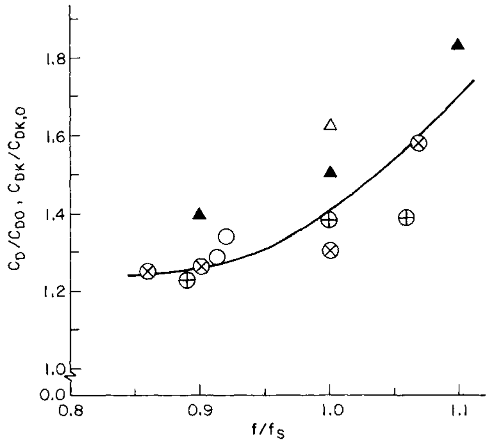

Fig. 2 The drag coefficient on a vibrating circular cylinder as a function of frequency f (normalized by the Strouhal frequency f_s) within the regime of wake capture or lock-in. ***Legend for data points:*** **Direct measurements C_D {Tanida, Okajima and Watanabe[5]}, $Re=80$, $C_{DO}=1.30$, ⊗; $Re=4000$, $C_{DO}=0.90$, ⊕; {Griffin, Skop and Koopmann[6]}, $Re=550$-900, $C_{DO}=0.91$, O; von Karman drag C_{DK} {Griffin and Ramberg[4]}, $Re=144$, $C_{DK,0}=1.21$, ▲; $Re=450$, $C_{DK,0}=0.90$, △. Cylinder amplitude, peak-to-peak = 0.28-0.30 diam (all cases).**

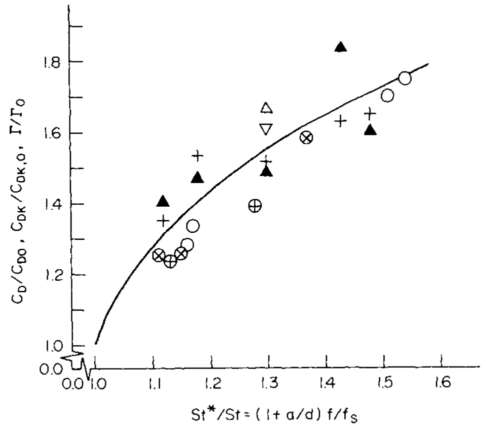

Fig. 3 The von Kármán drag coefficient C_{DK}, the vortex circulation Γ, and the drag coefficient from direct measurements C_D as a function of the wake capture Strouhal number St^*. ***Legend for data points:*** **von Kármán drag coefficient C_{DK}: see legend for Fig. 2. Vortex circulation Γ, same conditions as C_{DK}. Drag coefficient C_D, direct measurements {Griffin, Skop and Koopmann,[6] Tanida, Okajima, and Watanabe[5]}; see legend for Fig. 2.**

ther suggested, since the skin friction contribution is a small or negligible percentage of the total drag at subcritical Reynolds numbers between 50 and $2(10^5)$, that the drag on a bluff body is principally caused by $\Delta C_{D_{\text{other}}}$. However, the origin of the "other" contribution is not suggested. Recent published results[3-6] clearly show that the drag on stationary and vibrating cylinders at subcritical Reynolds numbers is largely the result of vortex formation in the bluff body wake, i.e., von Kármán drag.

To support this conclusion, the vortex strength, spacing, and viscous vortex core radius, simultaneously obtained from wake measurements, are listed in Table 1.[3-4] Also included in this table are the von Kármán drag coefficients C_{DK} obtained from the vortex strength and spacing by means of the equation[9-11]

$$C_{DK} = \pi\left[\frac{\Gamma}{\pi Ud}\right]\left\{\frac{\Gamma}{\pi UD}\right\}\frac{d}{\ell} + 2\left\{2\frac{f\ell}{U} - I\right\}\frac{h}{\ell}\Bigg] \qquad (11)$$

The stationary body results in Table 1 are in good agreement with the direct measurements given by Morkovin,[22] and the vibrating cylinder results are also in good agreement with other direct measurements. Tanida, Okajima and Watanabe[5] have measured the drag on a vibrating cylinder for a wide range of experimental conditions at Reynolds numbers of 80 and 4000. In their experiments a cylinder was towed through still water in a tank and vibrated transverse to the tow direction at frequencies within the lock-in regime. Several of the results from Table 1 are compared with some direct measurements in Fig. 2. The drag coefficients C_D on the vibrating cylinder are normalized by the stationary cylinder values to minimize any Reynolds number effects and to obtain a measure of the drag amplification at various frequencies within the lock-in regime. As luck would have it, the amplitudes of vibration for all of the experimental data in Fig. 2 are very nearly equal at about 30% of a diameter, which allows a straightforward comparison. Some drag measurements from the results of Griffin, Skop and Koopmann[6] are also included in the figure. These results were obtained from drag measurements on a cylinder which underwent vortex-induced oscillations in a wind tunnel at Reynolds numbers between 550 and 900. The figure shows reasonably good agreement between direct force measurements and the steady drag obtained from wake measurements of the vortex strength and spacing, and the drag amplification that accompanies the vibrations is clearly shown by all of these results.

Further evidence of this behavior is given in Fig. 3 where the von Kármán drag coefficient C_{DK} and the vortex circulation Γ are plotted with direct measurements of C_D as a function of the parameter St^*. Again there is a close correspondence between the direct force measurements and the drag and circulation which result from a wake velocity survey. The vortex strength (circulation), vortex street drag, and direct drag measurements generally fall on a single curve when the different Strouhal numbers St of the several experiments are taken into account. The Strouhal numbers for the stationary cylinders in both air and water ranged from 0.18-0.21 for the experiments performed at NRL, and from 0.14-0.18 for the experiments of Tanida et al.[5]

These results further confirm the close relationship between the wake response, as denoted by the vortex strength, and fluid forces, as denoted by the steady drag amplification, when a bluff body is vibrating under conditions of wake capture or lock-in. The vortex street drag is the primary contribution to the total drag at these Reynolds numbers as low as 80-450, and this is in disagreement with the suggestion by Payne that the vortex-induced drag contribution becomes negligible as the lower limit of the vortex shedding range of Reynolds numbers is approached.

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